

3.1-3 2. Linear Differential Equations of Higher Order

1. An Equation of the form

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_{n-1} \frac{dy}{dx} + P_n y = Q(x)$$

where P_1, P_2, \dots, P_n are all continuous and real valued functions of 'x' is called a linear differential equation of order 'n'.

2. An Equation of the form

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_{n-1} \frac{dy}{dx} + P_n y = Q(x)$$

where P_1, P_2, \dots, P_n are real constants and $Q(x)$ is a continuous function of 'x' is called an ordinary linear equation of order 'n' with constant co-efficients.

Roots of Auxiliary Equation $f(m) = 0$ and the complementary functions were as follows:-

$$\Rightarrow m = a, b, c$$

$$C.F. = c_1 e^{ax} + c_2 e^{bx} + c_3 e^{cx}$$

$$\Rightarrow * m = a, a$$

$$C.F. = (c_1 + c_2 x) e^{ax}$$

$$* m = b, b, b$$

$$C.F. = (c_1 + c_2 x + c_3 x^2) e^{bx}$$

$$\Rightarrow * m = a \pm ib$$

$$C.F. = e^{ax} [c_1 \cos bx + c_2 \sin bx]$$

$$* m = a \pm ib \text{ twice}$$

$$C.F. = e^{ax} [(c_1 + c_2 x) \cos bx + (c_3 + c_4 x) \sin bx]$$

$$\Rightarrow m = a, a, b$$

$$C.F. = (c_1 + c_2 x) e^{ax} + c_3 e^{bx}$$

Problems

1. Solve $2\frac{d^2y}{dx^2} - 13\frac{dy}{dx} + 15y = 0$

Soln Given Equation

$$2\frac{d^2y}{dx^2} - 13\frac{dy}{dx} + 15y = 0$$

$$D = \frac{d}{dx}$$

$$(2D^2 - 13D + 15)y = 0$$

Auxiliary Equation

$$2m^2 - 13m + 15 = 0$$

$$2m^2 - 10m - 3m + 15 = 0$$

$$2m(m-5) - 3(m-5) = 0$$

$$(2m-3)(m-5) = 0$$

$$m = 5, m = \frac{3}{2}$$

$$C.F \Rightarrow y = C_1 e^{5x} + C_2 e^{\frac{3}{2}x}$$

2. Solve $(D^2 - 4D + 4)y = 0$

Soln Given Equation

$$(D^2 - 4D + 4)y = 0$$

Auxiliary Equation

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$C.F \quad y = (C_1 + C_2 x) e^{2x}$$

3. Solve $\frac{d^2y}{dx^2} + 9y = 0$

Soln Given Equation

$$\frac{d^2y}{dx^2} + 9y = 0$$

$$(D^2 + 9)y = 0$$

Auxiliary Equation

4 Solve $(D^2 - 4D + 2)y = 0$

Solu Given Equation

$$(D^2 - 4D + 2)y = 0$$

Auxiliary Equation

$$m^2 - 4m + 2 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{16 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$= 2 \pm \sqrt{2}$$

$$C.F = c_1 e^{(2+\sqrt{2})x} + c_2 e^{(2-\sqrt{2})x}$$

5 Solve $(D^3 + 3D^2 - 4)y = 0$

Solu Given Equation

$$(D^3 + 3D^2 - 4)y = 0$$

Auxiliary Equation

$$m^3 + 3m^2 - 4 = 0$$

$$m = 1, \Rightarrow 1 + 3 - 4 = 4 - 4 = 0$$

$$\begin{array}{r|l} m-1 & m^3 + 3m^2 - 4 \\ & m^3 - m^2 \\ \hline & 4m^2 - 4 \\ & 4m^2 - 4m \\ \hline & 4m - 4 \\ & 4m - 4 \\ \hline & 0 \end{array} \quad m^2 + 4m + 4$$

$$(m-1)(m^2 + 4m + 4) = 0$$

$$(m-1)(m+2)^2 = 0$$

$$m = 1, -2, -2$$

int220.blogspot.com

Scanned by CamScanner

$$C.F = y = c_1 e^x + (c_2 + c_3 x) e^{-2x}$$

6 solve $(D^4 + 8D^2 + 16)y = 0$

Solu Given Equation

$$(D^4 + 8D^2 + 16)y = 0$$

$$C.F = y = c_1 e^x + (c_2 + c_3 x) e^{-x}$$

6 solve $(D^4 + 8D^2 + 16)y = 0$

Solu Given Equation

$$(D^4 + 8D^2 + 16)y = 0$$

Auxiliary Equation

$$m^4 + 8m^2 + 16 = 0$$

$$(m^2 + 4)^2 = 0$$

$$m^2 + 4 = 0$$

$$m = \pm 2i \text{ twice}$$

$$C.F = (c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x$$

H.W

7 $(D^3 - 2D^2 - 3D)y = 0$

8 $(D^3 - 14D + 8)y = 0$

9 $(D^2 + 25)y = 0$

10 $(D^2 - 16)y = 0$

11 $(4D^3 + 4D^2 + D)y = 0$

12 $(D^4 + 18D^2 + 81)y = 0$

13 $\frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = 0$

14 $\frac{d^3 y}{dx^3} - \frac{9d^2 y}{dx^2} + 23 \frac{dy}{dx} - 15y = 0$

15 $(D^3 - 3D^2 + 2)y = 0$

16 $(D^2 + 15D + 24)y = 0$

7 $(D^3 - 2D^2 - 3D)y$

Auxiliary Equation

$$m^3 - 2m^2 - 3m = 0$$

$$m^3 - 3m^2 + m^2 - 3m = 0$$

$$m^2(m-3) + m(m-3) = 0$$

$$(m^2 + m)(m-3) = 0$$

$$m^2 = -m \quad m = 3$$

$$m = \pm 1$$

$$C.F \Rightarrow y = c_1 e^x + c_2 e^{3x} + (c_1 + c_2)x e^x + c_3 e^{3x}$$

$$\textcircled{1} \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = e^{2x}$$

Solu

Given Equation

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = e^{2x}$$

$$(D^2 + 4D + 3)y = e^{2x}$$

Auxiliary Equation

$$m^2 + 4m + 3 = 0$$

$$(m+3)(m+1) = 0$$

$$m = -1, -3$$

Scanned by CamScanner

$$C.F = c_1 e^{-x} + c_2 e^{-3x}$$

$$P.I = \frac{e^{2x}}{D^2 + 4D + 3} = \frac{e^{2x}}{2^2 + 4(2) + 3}$$

$$= \frac{e^{2x}}{4 + 8 + 3} = \frac{e^{2x}}{15}$$

$$\text{G.S } y = C.F + P.I$$

$$y = c_1 e^{-x} + c_2 e^{-3x} + \frac{e^{2x}}{15}$$

$$2. (D^2 - 4D + 13)y = e^{2x} + e^x$$

Solu

Given Equation

$$(D^2 - 4D + 13)y = e^{2x} + e^x$$

Auxiliary Equation

$$m^2 - 4m + 13 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{16 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$3. (D^2 + 2D + 1)y = e^x + e^{-x}$$

Soln) Given Equation

$$(D^2 + 2D + 1)y = e^x + e^{-x}$$

Auxiliary Equation

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m = -1, -1$$

$$C.F = (c_1 + c_2 x)e^{-x}$$

$$P.I = \frac{e^x}{D^2 + 2D + 1} + \frac{e^{-x}}{D^2 + 2D + 1}$$

$$= \frac{e^x}{1+2+1} + \frac{e^{-x}}{(D+1)^2}$$

$$= \frac{e^x}{4} + \frac{e^{-x}}{(D+1)^2}$$

$$= \frac{e^x}{4} + \frac{e^{-x}}{(D+1)^2}$$

$$= \frac{e^x}{4} + \frac{e^{-x}}{D^2}$$

$$= \frac{e^x}{4} + e^{-x} \frac{1}{D} \int dx$$

$$= \frac{e^x}{4} + e^{-x} \frac{1}{D} x$$

$$= \frac{e^x}{4} + e^{-x} \int x dx$$

$$P.I = \frac{e^x}{4} + e^{-x} \frac{x^2}{2}$$

$$G.S = C.F + P.I$$

$$= (c_1 + c_2 x)e^{-x} + \frac{e^x}{4} + e^{-x} \frac{x^2}{2}$$

$$4. (4D^2 - 4D + 1)y = 100$$

Soln) Given Equation

$$m = 1/2, \frac{1}{2}$$

$$C.F = (c_1 + c_2 x) e^{1/2 x}$$

$$P.I = \frac{100}{4D^2 + 4D + 1} = \frac{100}{(2D+1)^2} = \frac{100}{1} = 100$$

$$y = C.F + P.I$$

$$= (c_1 + c_2 x) e^{1/2 x} + 100$$

5. Hw $(D^2 - 3D + 2)y = \cosh x$

Solu Given Equation

$$(D^2 - 3D + 2)y = \cosh x = \frac{e^x + e^{-x}}{2}$$

Auxiliary Equation

$$m^2 - 3m + 2 = 0$$

$$m^2 - 2m - m + 2 = 0$$

$$m(m-2) - 1(m-2) = 0$$

$$(m-1)(m-2) = 0$$

$$m = 1, 2$$

$$C.F = c_1 e^x + c_2 e^{2x}$$

$$P.I = \frac{1}{2} \left[\frac{e^x + e^{-x}}{2} \right]$$

$$= \frac{1}{2} \left[\frac{e^x}{D^2 - 3D + 2} + \frac{e^{-x}}{D^2 - 3D + 2} \right]$$

$$= \frac{1}{2} \left[\frac{e^x}{D^2 - 3D + 2} + \frac{e^{-x}}{1 + 3 + 2} \right]$$

$$= \frac{1}{2} \left[\frac{e^x}{D^2 - 3D + 2} + \frac{e^{-x}}{6} \right]$$

Scanned by C

$$= \frac{1}{2} \left[\frac{e^x}{(D-1)(D-2)} + \frac{e^{-x}}{6} \right]$$

$$= \frac{1}{2} \left[\frac{e^x}{(D-1+1)(D-2)} + \frac{e^{-x}}{6} \right]$$

$$= \frac{1}{2} \left[\frac{e^x}{D(D-2)} + \frac{e^{-x}}{6} \right]$$

1. Solve $(D^2 + 3D + 2)y = \sin 3x$

Solu $(D^2 + 3D + 2)y = \sin 3x$

Auxiliary Equation

$$m^2 + 3m + 2 = 0$$

$$(m+2)(m+1) = 0$$

$$m = -2, m = -1$$

$$C.F = c_1 e^{-2x} + c_2 e^{-x}$$

$$P.I = \frac{\sin 3x}{D^2 + 3D + 2}$$

$$= \frac{\sin 3x}{-9 + 3D + 2} = \frac{\sin 3x}{3D - 7}$$

$$= \frac{(3D + 7) \sin 3x}{(3D + 7)(3D - 7)}$$

$$= \frac{(3D + 7) \sin 3x}{9D^2 - 49}$$

$$= \frac{(3D + 7) \sin 3x}{9(-9) - 49}$$

$$= -\frac{1}{130} \{ 3(3 \cos 3x) + 7 \sin 3x \}$$

$$= -\frac{1}{130} \{ 9 \cos 3x + 7 \sin 3x \}$$

$$y = C.F + P.I$$

$$= c_1 e^{-2x} + c_2 e^{-x} - \frac{1}{130} (9 \cos 3x + 7 \sin 3x)$$

2. $(D^2 - 4)y = 2 \cos^2 x$

Solu Given

$$(D^2 - 4)y = 2 \cos^2 x$$

$$= 1 + \cos 2x$$

Auxiliary Equation

$$m^2 - 4 = 0$$

$$\Rightarrow (m+2)(m-2) = 0$$

$$m = 2, m = -2$$

$$C.F = c_1 e^{-2x} + c_2 e^{2x}$$

$$\begin{aligned} 1 + \cos 2x &= 2 \cos^2 x \\ 2 \cos^2 x - 1 &= 1 \\ 2 \cos^2 x &= 2 \\ \cos^2 x &= 1 \end{aligned}$$

$$\Rightarrow (m+2)(m-2) = 0$$

$$m = 2, m = -2$$

$$C.F = c_1 e^{-2x} + c_2 e^{2x}$$

$$P.I = \frac{1 + \cos 2x}{D^2 - 4}$$

$$= \frac{1}{D^2 - 4} + \frac{\cos 2x}{D^2 - 4}$$

$$= \frac{1}{0 - 4} + \frac{\cos 2x}{-4 - 4}$$

$$= -\frac{1}{4} - \frac{1}{8} \cos 2x$$

$$y = C.F + P.I$$

$$= c_1 e^{-2x} + c_2 e^{2x} - \frac{1}{4} - \frac{1}{8} \cos 2x$$

Scanned by CamScanner

$$3. (D^2 - 4D + 3)y = \sin 3x \cos 2x$$

Sol Given

$$[(D^2 - 4D + 3)y = 2 \cos^2 x]$$

$$= 1 + \cos 2x$$

Auxiliary equation

$$m^2 - 4m + 3 = 0$$

$$(m+2)(m-2) = 0$$

$$m = 2, -2$$

$$C.F = c_1 e^{-2x} + c_2 e^{2x}$$

$$P.I = \frac{1 + \cos 2x}{D^2 - 4}$$

$$= \frac{1}{D^2}$$

$$C.F = c_1 e^x + c_2 e^{3x}$$

$$P.I = \frac{1}{2} \left[\frac{\sin 5x}{D^2 - 4D + 3} + \frac{\sin x}{D^2 - 4D + 3} \right]$$

$$= \frac{1}{2} \left[\frac{\sin 5x}{-25 - 4D + 3} + \frac{\sin x}{-1 - 4D + 3} \right]$$

Given

$$(D^2 - 4D + 3)y = \sin 3x \cos 2x$$

$$= \frac{1}{2} [2 \sin 3x \cos x]$$

$$= \frac{1}{2} [\sin 5x + \sin x]$$

$$\therefore \sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

Auxiliary equation

$$m^2 - 4m + 3 = 0$$

$$(m-1)(m-3) = 0$$

$$m = 1, 3$$

$$\begin{matrix} 3 \\ -1-3 \\ \hline -1-3 \end{matrix}$$

$$\begin{aligned}\cos(A+B) + \cos(A-B) &= 2\cos A \cos B \\ \sin(A+B) - \sin(A-B) &= 2\cos A \sin B \\ \sin(A-B) + \sin(A+B) &= 2\sin A \cos B \\ \sin(A+B) + \sin(A-B) &= 2\sin A \cos B\end{aligned}$$

Auxiliary Equation

$$m^2 + 5m - 6 = 0$$

$$m^2 + 6m - m - 6 = 0$$

$$m(m+6) - 1(m+6) = 0$$

$$(m-1)(m+6) = 0$$

$$m = 1, m = -6$$

$$C.F. = C_1 e^x + C_2 e^{-6x}$$

$$P.I. = \frac{1}{2} \left[\frac{\cos 3x}{D^2 + 5D - 6} - \frac{\cos 5x}{D^2 + 5D - 6} \right]$$

$$= \frac{1}{2} \left[\frac{\cos 3x}{-9 + 5D - 6} - \frac{\cos 5x}{-25 + 5D - 6} \right]$$

$$= \frac{1}{2} \left[\frac{\cos 3x}{5D - 15} - \frac{\cos 5x}{5D - 31} \right]$$

$$= \frac{1}{2} \left[\frac{(5D+15)\cos 3x}{(5D+15)(5D-15)} - \frac{\cos 5x(5D+31)}{(5D+31)(5D-31)} \right]$$

$$= \frac{1}{2} \left[\frac{(5D+15)\cos 3x}{(5D)^2 - (15)^2} - \frac{\cos 5x(5D+31)}{(5D)^2 - (31)^2} \right]$$

$$= \frac{1}{2} \left[\frac{(5D+15)\cos 3x}{25(-9) - 225} - \frac{\cos 5x(5D+31)}{25(-25) - 961} \right]$$

$$= \frac{1}{2} \left[\frac{(5D+15)\cos 3x}{-225 - 225} - \frac{\cos 5x(5D+31)}{-625 - 961} \right]$$

$$= + \frac{1}{2} \left[\frac{(5D+15)\cos 3x}{-450} - \frac{\cos 5x(5D+31)}{-1586} \right]$$

$$= - \frac{1}{2} \left[\frac{(5D+15)\cos 3x}{450} - \frac{\cos 5x(5D+31)}{1586} \right]$$

$$= - \frac{1}{2} \left[\frac{-5 \sin 3x(3) + 15 \cos 3x}{450} - \frac{5(-\sin 5x) \cdot 5 + 31 \cos 5x}{1586} \right]$$

$$= - \frac{1}{2} \left[\frac{31 \cos 3x - 15 \sin 3x}{450} - \frac{31 \cos 5x - 25 \sin 5x}{1586} \right]$$

$$\begin{array}{r} 31 \\ 25 \overline{) 31} \\ \underline{7} \\ 225 \overline{) 22593} \\ \underline{961} \\ 625 \\ \underline{1586} \\ 215 \\ \underline{225} \\ 950 \end{array}$$

$$y = c_1 e^x + c_2 e^{-x} - \frac{1}{2} e^{-\frac{x}{2}}$$

$$(D^2 - 2D + 2)y = \cos 9x$$

Auxiliary Equation

$$m^2 - 2D + 2 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2}$$

$$= \frac{2(1 \pm i)}{2} = 1 \pm i$$

$$C.F = e^x [(c_1 e^{ix}) \cos x + (c_2 + k_1 x) \sin x]$$

$$P.I = \frac{\cos 9x}{D^2 - 2D + 2}$$

$$= \frac{\cos 9x}{-81 - 2D + 2} = \frac{\cos 9x}{-2D - 79} = \frac{\cos 9x}{-(2D + 79)}$$

$$= - \left[\frac{\cos 9x (2D - 79)}{(2D - 79)(2D + 79)} \right]$$

$$= - \left[\frac{\cos 9x (2D - 79)}{(2D)^2 - (79)^2} \right]$$

$$= - \left[\frac{\cos 9x (2D - 79)}{4D^2 - 6241} \right]$$

$$= - \left[\frac{2D \cos 9x - 79 \cos 9x}{4(-81) - 6241} \right]$$

$$= - \left[\frac{2(-\sin 9x)9 - 79 \cos 9x}{-324 - 6241} \right]$$

$$= - \left[\frac{-18 \sin 9x - 79 \cos 9x}{6565} \right]$$

$$= - \left[\frac{-(18 \sin 9x + 79 \cos 9x)}{6565} \right] = \left[\frac{18 \sin 9x + 79 \cos 9x}{6565} \right]$$

$$\begin{array}{r} 79 \\ 79 \quad 862 \\ \hline 711 \quad 711 \\ 591 \quad 553 \\ \hline 642124 \end{array}$$

$$\begin{array}{r} 81 \\ 4 \\ \hline 324 \\ 6241 \\ 324 \\ \hline 6565 \end{array}$$

$$\frac{18}{25}$$

Scanned by CamSc

$$y = C.F + P.I$$

$$y = e^x [(c_1 e^{ix}) \cos x + (c_2 + k_1 x) \sin x + \frac{18 \sin 9x + 79 \cos 9x}{6565}]$$

7. Given

$$y = C.F + P.I$$

$$= c_1 e^x + c_2 e^{2x} + \frac{1}{2} \left[\frac{\cos x - 3 \sin x}{10} - \left(\frac{23 \cos 5x + 3 \sin 5x}{754} \right) \right]$$

11. Given that

$$(D^2 + D)y = \cos x + e^{2x}$$

Auxiliary Equation.

$$m^2 + m = 0$$

$$m(m+1) = 0$$

$$m = 0; m+1 = 0$$

$$m = -1$$

$$C.F = c_1 e^{0x} + c_2 e^{-x} = c_1 + c_2 e^{-x}$$

$$P.I = \frac{\cos x}{D^2 + D} + \frac{e^{2x}}{D^2 + D}$$

$$= \frac{\cos x}{-1 + D} + \frac{e^{2x}}{2^2 + 2}$$

$$= \frac{\cos x}{D-1} + \frac{e^{2x}}{4+2}$$

$$= \frac{(\cos x)(D+1)}{(D+1)(D-1)} + \frac{e^{2x}}{6}$$

$$= \frac{\cos x(D+1)}{D^2 - 1^2} + \frac{e^{2x}}{6}$$

$$= \frac{D \cos x + \cos x}{D^2 - 1} + \frac{e^{2x}}{6}$$

$$= \frac{D \cos x + \cos x}{-1-1} + \frac{e^{2x}}{6}$$

$$= \frac{-\sin x + \cos x}{-2} + \frac{e^{2x}}{6}$$

$$= \frac{\cos x - \sin x}{-2} + \frac{e^{2x}}{6}$$

$$P.I = \frac{\sin x - \cos x}{2} + \frac{e^{2x}}{6}$$

Scanned by CamS

$$y = C.F + P.I$$

$$y = c_1 + c_2 e^{-x} + \frac{\sin x - \cos x}{2} + \frac{e^{2x}}{6}$$

15. Given that

$$\begin{aligned}(D^2 - 3D + 2)y &= \cos 3x \cdot \cos 2x \\ &= \frac{1}{2} [2 \cos 3x \cdot \cos 2x] \\ &= \frac{1}{2} [\cos x + \cos 5x]\end{aligned}$$

Auxiliary Equation

$$\begin{aligned}m^2 - 3m + 2 &= 0 \\ m^2 - 2m - m + 2 &= 0 \\ m(m-2) - 1(m-2) &= 0 \\ (m-1)(m-2) &= 0 \\ m=1, m=2\end{aligned}$$

$$C.F = C_1 e^x + C_2 e^{2x}$$

$$\begin{aligned}P.I &= \frac{1}{2} \left[\frac{\cos x}{D^2 - 3D + 2} + \frac{\cos 5x}{D^2 - 3D + 2} \right] \\ &= \frac{1}{2} \left[\frac{\cos x}{-1 - 3D + 2} + \frac{\cos 5x}{-25 - 3D + 2} \right] \\ &= \frac{1}{2} \left[\frac{\cos x}{1 - 3D} + \frac{\cos 5x}{-(3D + 23)} \right] \\ &= \frac{1}{2} \left[\frac{\cos x}{1 - 3D} - \frac{\cos 5x}{3D + 23} \right] \\ &= \frac{1}{2} \left[\frac{(1+3D)\cos x}{(1+3D)(1-3D)} - \frac{(3D-23)\cos 5x}{(3D+23)(3D-23)} \right] \\ &= \frac{1}{2} \left[\frac{(1+3D)\cos x}{1-9D^2} - \frac{(3D-23)\cos 5x}{9D^2 - 529} \right] \\ &= \frac{1}{2} \left[\frac{(1+3D)\cos x}{1-9(-1)} - \frac{(3D-23)\cos 5x}{9(-25) - 529} \right] \\ &= \frac{1}{2} \left[\frac{\cos x + 3D\cos x}{10} + \frac{3D\cos 5x - 23\cos 5x}{-225 - 529} \right]\end{aligned}$$

$$\begin{array}{r}23 \\ 23 \\ \hline 46 \\ 529 \\ \hline 575 \\ 225 \\ \hline 754\end{array}$$

$$\begin{aligned}&= \frac{1}{2} \left[\frac{\cos x + 3D\cos x}{10} + \frac{3D\cos 5x - 23\cos 5x}{754} \right] \\ &= \frac{1}{2} \left[\frac{\cos x - 3\sin x}{10} + \left(\frac{23\cos 5x + 3\sin 5x}{754} \right) \right]\end{aligned}$$

$$y = C.F + P.I$$

$$y = C_1 e^x + C_2 e^{2x} + \left[\frac{\cos x - 3\sin x}{10} + \frac{23\cos 5x + 3\sin 5x}{754} \right]$$

Scanned by CamScanner

14. Given that
 $(D^3+1)y = \sin(2x+1)$
 Auxiliary Equation
 $m^3+1=0$
 $m^3+1^3=0$

$$(m+1)(m^2-m+1)=0$$

$$\left. \begin{array}{l} m+1=0 \\ m=-1 \end{array} \right\} \begin{array}{l} m^2-m+1=0 \\ m = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \end{array}$$

$$= \frac{+1 \pm \sqrt{1-4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{3}i}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$C.F = c_1 e^{-x} + e^{1/2x} \left[c_2 \cos \frac{\sqrt{3}}{2}x + c_3 \sin \frac{\sqrt{3}}{2}x \right]$$

$$P.I = \frac{\sin(2x+1)}{D^3+1} \Rightarrow \frac{\sin(2x+1)}{-2D+1} \Rightarrow \frac{\sin(2x+1)}{1-2D}$$

$$= \frac{[\sin(2x+1)](1+2D)}{(1-2D)(1+2D)} \Rightarrow \frac{[\sin(2x+1)](1+2D)}{1^2-(2D)^2}$$

$$= \frac{[\sin(2x+1)](1+2D)}{1-4D^2}$$

$$= \frac{[\sin(2x+1)](1+2D)}{1-4D^2}$$

$$= \frac{[\sin(2x+1)](1+2D)}{1-4D^2} = \frac{[\sin(2x+1)](1+2D)}{69}$$

$$= \frac{\sin(2x+1) + 2D \sin(2x+1)}{69}$$

Scanned by CamScanner

$$= + \left[\frac{y \cos(2x+1) + \sin(2x+1)}{75} \right]$$

$$y = C.F + P.I$$

$$= c_1 e^{-x} + e^{1/2x} \left[c_2 \cos \left(\frac{\sqrt{3}}{2} \right) x + c_3 \sin \frac{\sqrt{3}}{2} x \right] + \left[\frac{y \cos(2x+1) + \sin(2x+1)}{69} \right]$$

$$4 \quad (D^2+16)y = \sin 3x + \cos 2x$$

Solu

Given

$$(D^2+16)y = \sin 3x + \cos 2x$$

Auxiliary Equation

$$m^2+16=0$$

$$m = \pm 4i$$

$$C.F = C_1 \cos 4x + C_2 \sin 4x$$

$$P.I = \frac{\sin 3x}{D^2+16} + \frac{\cos 2x}{D^2+16}$$

$$= \frac{\sin 3x}{-9+16} + \frac{\cos 2x}{-4+16}$$

$$= \frac{\sin 3x}{7} + \frac{\cos 2x}{12}$$

$$y = C.F + P.I$$

$$= C_1 \cos 4x + C_2 \sin 4x + \frac{\sin 3x}{7} + \frac{\cos 2x}{12}$$

$$5. \text{ solve } (D^2+5D-6)y = \sin x \cos x$$

$$6. \text{ solve } (D^2-2D+2)y = \cos 9x$$

$$7. \text{ solve } (D^2+1)y = \sin x \sin 2x$$

$$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

Solu Given that

$$(D^2+5D-6)y = \sin x \sin x$$

$$= \frac{1}{2} [2 \sin x \sin x]$$

$$= \frac{1}{2} [\cos 3x - \cos 5x]$$

Formulae

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B \checkmark$$

Scanned by CamScanner

$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$\cos(A-B) + \cos(A+B) = 2 \cos A \cos B$$

$$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$$

$$\sin(A-B) + \sin(A+B) = 2 \sin A \cos B$$

$$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$$